# ENTRANCE EXAMINATION TO THE CYCLES OF TECHNICIAN SUPERIOR AND TECHNICIAN OF THE AFRICAN SCHOOL OF THE METEOROLOGY AND THE CIVIL AVIATION (EAMAC)

**SESSION 2013**

TEST OF : MATHEMATICS

DURATION : 3 HOURS

**Exercise 1** ($5pts$)

One considers in $\mathbb{C}$ the sequence of general term $z_n$ defined by:

$$\begin{cases} z_0 = 1 \\ 2z_{n+1} = z_n + i \end{cases}$$

1. Show that, for each integer $n$, no vanishing, the module $r_n$ of $z_n$ is lower than 1.

2. We set $z_n = x_n + iy_n$, $x_n$ and $y_n$ being real numbers and $u_n = z_n - i$
a. Find a recurrence relation between $u_{n+1}$ and $u_n$.
b. Deduce that the sequence $(x_n)$ is a geometric sequence which converges towards 0 and the sequences of general terms $y_n$ and $r_n$ converge towards 1.

3. Calculate the minor term $n_0$ such that, for each $n$ equal or higher than $n_0$, one has : $|z_n - i| < 10^{-5}$.

**Exercise 2** ($4pts$)

1. Solve the differencial equation : $9y'' + 4y = 4\sqrt{3}$.

2. a. Determine among the solutions of this equation the solution $h$ such that : $h'\left(\frac{\pi}{2}\right) = \dfrac{4}{3}$ and $h''\left(\frac{\pi}{4}\right) = \dfrac{4}{9}$.
b. Write $h(x)$ in the form $A + B\cos(\omega x + \varphi)$ where $A, B, \omega$ and $\varphi$ are four reales that one will specify.

3. Solve in $\mathbb{R}$ the equation $h(x) = 0$.

**Exercise 3** ($5pts$)

$f$ is the numerical function defined on $\mathbb{R}_+^*$ by : $f(x) = \ln\left[\dfrac{e}{2}\left(x + \dfrac{1}{x}\right)\right]$

One calls $(C)$ the representative curve of $f$ in an orthonormal reference $\left(O, \vec{i}, \vec{j}\right)$.

1. a. Study $f$, then show that $f$ has a minimum; that one must specify

2. b. Show that the curve $(\Gamma)$ of equation $y = \ln\left(\frac{e}{2}x\right)$ is an asymptote of the curve $(C)$.

c. Draw the curves $(C)$ and $(\Gamma)$ in the same orthonormal reference $\left(O, \vec{i}, \vec{j}\right)$.

3. a. Establish that for any $x$ in $\mathbb{R}_+^*$, $f'(x) < 1$

b. Deduce the sign of $(f(x) - x)$ and the position of the curve $(C)$ according to the line $(D)$ of equation $y = x$

4. a. Deduce from previous results that the sequence $(u_n)$ which verifies :

$$\begin{cases} u_0 \in \mathbb{R}_+^* \\ u_{n+1} = f(u_n) = \ln\left[\frac{e}{2}\left(u_n + \frac{1}{u_n}\right)\right] \end{cases}$$

is decreasing and undervalued by 1 for starting by row $n = 1$ (for starting by row $n = 0$ if $u_0 \geq 1$ ).

b. Show that the sequence $(u_n)$ converges towards 1.

**Exercise 4** (6*pts*)

1. A ballot box contains two white balls and $n$ black balls, indistinguishable by touch.
A player extracts simultaneously two balls from the ballot box and one notes $A_2$ the event:
$A_2$ : « the player extracted two white balls ».
Determine $n$ so that the probability of $p(A_2)$ is equal to $1/15$.
**In the following part of the exercise, we will take $n = 4$.**

2. A player extracts simultaneously two balls from the ballot box and one notes :
$A_0$: « The player extracts extracts two black balls » ;
$A_1$ : « The player extracts one black ball and one white ball» ;
$A_2$ : « The player extracts extracts two white balls».
a. Calculate $p(A_0)$ and $p(A_1)$.
b. With this pulling, the player gets three points for each white ball extracted and two points for each black ball extracted. Let $X$ be the random variable associated to the numbers of points obtained.
Determine the law of probability of the random variable $X$ and calculate its mean $E(X)$.

3. After this first pulling, the player drawn back the black balls and leaves the white ones, then extracts simultaneously two balls from the ballot box.
a. Give $p(B_0/A_2)$ and deduce $p(B_0 \cap A_2)$ ; Calculate $p(B_0/A_1)$ and $p(B_0 \cap A_1)$.
Deduce that $p(B_0) = \frac{41}{75}$.
b. Show that $p(B_2) = \frac{2}{75}$. Deduce $p(B_1)$.